Supplemental Material: Long-range anomalous decay of the correlation in jammed packings

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INTRODUCTION TO THE GENERAL PAIR CORRELATION FUNCTION

Let us first recall the equations introduced in the main text. Given a generic observable $\mathcal{O}$, let us define the correlation function between particles i and j at positions $\mathbf{r}_i$ and $\mathbf{r}_j$ respectively, by rewriting the RDF as the correlation function between particles i and j at positions $\mathbf{r}_i$ and $\mathbf{r}_j$ respectively, as

$$g^s_{\mathcal{O}}(r) = \frac{1}{C}\sum_{i,j}\delta(|\mathbf{r}_i - \mathbf{r}_j| - r)\mathcal{O}^s_i\mathcal{O}^s_j\,, \tag{S1}$$

where r is the distance between the particles i and j, C is the normalization factor and $s \in \Re$ a control parameter. Notice that for $s > 0$ the main contribution to (S1) comes from large values of $\mathcal{O}$, for $s < 0$ the correlation between small values of $\mathcal{O}$ is enhanced and for $s = 0$ the usual expression for the RDF, i.e. $g^{s=0}_{\mathcal{O}}(r) \equiv g(r)$, is recovered. However, (S1) not only contains information about the point-to-point correlation of the observable $\mathcal{O}$ but also has a contribution due to the variation in the displacement of the particles through the system. This density-density correlation appears as periodic oscillations decreasing with the distance r. In order to suppress this added contribution let us define the *generalized correlation function*

$$C^s_{\mathcal{O}}(r) = \frac{g^s_{\mathcal{O}}(r)}{g(r)}\,, \tag{S2}$$

where $g^s_{\mathcal{O}}(r)$ is given in (S1) and $g(r)$ is the classic RDF accounting only for the density-density correlation. In this work we analyze the correlation of the forces between particles at jamming and the correlation of the variation of the coordination number w.r.t the isostaticity, ΔZ.

RDF MID AND LONG-RANGE BEHAVIOR

We showed (see main text) that by introducing a system description w.r.t the contact points between particles it is possible to point out RDF features in the mid and long-range that are hidden by the usual system representation. To do this, let us note that the correlation decays with a behavior $\propto e^{-ar/\sigma'}$ with $a = 0.318 \pm 0.009$. The study of the function $g(r)e^{ar}$ evidenced that the RDF results by the superimposition of two different oscillating signals so that

$$g(r) \propto e^{-ar}\sum_{m,l} f_i(r) = e^{-ar}\sum_{m,l} c_i e^{-a_i r}\cos(p_i r + \psi_i) \tag{S3}$$

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where c_i, a_i, p_i, ψ_i are the function parameters and m, l denote the oscillations in the mid and long-range, respectively. In Table I we reported the complete set of parameters obtained by fitting the RDF according to (S3). Notice that $p_m \approx 2p_l = 7.514 \pm 0.003$ and $a_m \approx 2a_l = 0.70 \pm 0.01$ supporting the hypothesis that $f_l(r)$ can be considered as an $\mathcal{O}(2)$ correction to the $\mathcal{O}(1)$ mid-range leading term $f_m(r)$. Moreover, given that $c_m >> c_l$, the long-range oscillations get easily masked by the mid-range ones. Interestingly, the introduction of the fictive particles doubles the system size making it possible to observe both $f_m(r)$ and $f_l(r)$. This is not possible for the real particles system, where only the mid-range oscillations can be measured.

	$\mathbf{c_i}$	$\mathbf{a + a_i}$	$\mathbf{p_i}$	$\boldsymbol{\psi_i}$
m	1.26 (4)	0.70 (1)	7.514 (3)	-7.68 (2)
l	-0.046 (1)	0.31 (1)	3.805 (3)	-8.00 (2)

TABLE I. Complete set of parameters obtained by fitting the contacts RDF to (S3). m, l denote the mid and long-range behavior, respectively. The error of each parameter is shown in parenthesis. Note that the total damping coefficients $a'_i = a + a_i$ as a consequence of the procedure adopted to analyze the $g(r)$.

GEOMETRY OF THE FORCES NETWORK

When a system of monodisperse HS reaches the jamming point, the particles form a rigid network of contacts which is stabilized by the total balance of the forces exchanged in the contact points. Within this picture, is it possible to distinguish between *strong* and *weak* forces. In our work, we analyzed the weak forces originated by slight unbalances of the interactions between neighbouring particles. In Fig.S1 we schematically show the case of three HS in contact in $d = 2$. When the particles lie on the same plane, i.e. are perfectly centrosymmetric, the exchanged force is coplanar and perfectly balanced so that no other particle can be part of the force network (Fig.S1(a)). By contrast, if the middle sphere is out-of-plane the resulting force exhibits a small sideways component, which is the weak force (Fig.S1(b)): the higher the coplanarity, the smaller the weak force.

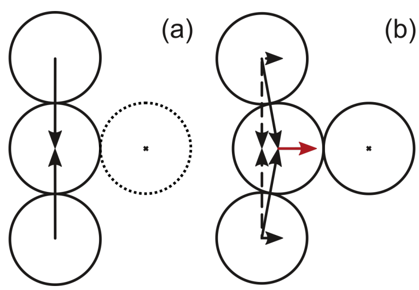

FIG. S1. Schematic representation of the interparticle forces in $d = 2$. **(a)** For particles lying in the same plane the forces are perfectly balanced (black arrows). No other particle is involved in the interaction and within this picture this is a strong force belonging to the main network. **(b)** When the particles are not coplanar, the extra components of the force constitute a weak interaction with another particle (red arrow).